



Coupled industrial production and energy supply planning PGMO Days

Zoé Fornier November 2022

- **2** Problem description
- 3 Operational problem solution methods
- **4** Design problem solution methods
- **5** Numerical Results

















CONTEXT



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Operational Problem



Design Problem



2 Problem description

Design Problem Formulation

Operational Problem Formulation

Operational problem solution methods

4 Design problem solution methods

6 Numerical Results

6 Conclusion

Design Problem Formulation

$$(\mathsf{P}): \quad \min_{\theta \in \Theta} \quad I(\theta) + V(x_0; \theta)$$

- Design variables: $\theta := \{v_t^{\mathsf{DA}}\}_{t \in [\mathcal{T}]}$
- Design constraints: $\Theta := \{v_t^{\mathsf{DA}} \ge 0, \forall t \in [T]\}$

Design Problem Formulation

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- Design variables: $\theta := \{v_t^{\mathsf{DA}}\}_{t \in [T]}$
- Design constraints: $\Theta := \{v_t^{\mathsf{DA}} \ge 0, \forall t \in [\mathcal{T}]\}$
- Design cost: $I(\theta) = \sum_{t=1}^{T} p_t^{DA} v_t^{DA}$;
- Parametrized problem cost: $V(x_0, \theta) := v(P_{\theta});$

Problem description Design Problem Formulation Operational Problem Formulation

Operational problem solution methods

- 4 Design problem solution methods
- **6** Numerical Results

6 Conclusion

Stochastic parametrized operational problem

$$(\mathsf{P}_{\theta}) \min_{(\boldsymbol{u}_{t},\boldsymbol{x}_{t})_{t}\in[\mathcal{T}]} \mathbb{E}\left[\sum_{t=1}^{I} L_{t}^{\theta}(\boldsymbol{x}_{t-1},\boldsymbol{u}_{t},\boldsymbol{q}_{t}^{\mathsf{PV}})\right]$$

• State variables: $\mathbf{x}_t := (SOC_t, s_t^1, s_t^2, s_t^3)$,

• Controls:
$$\boldsymbol{u}_t := (\underbrace{q_t^{\text{grid}}, v_t^{\text{ID}}, \phi_t^+, \phi_t^-, (u_t^{ij})_{i \in I, j \in J}}_{\in \mathbb{R}^+}, (\underbrace{b_t^{ij})_{i \in I, j \in J}}_{\in \{0,1\}}),$$

• Random variables : q_t^{PV} assumed independent.

Stochastic parametrized operational problem

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s.c $x_{t} = \mathsf{D}_{t}^{\theta}(\boldsymbol{x}_{t-1},\boldsymbol{u}_{t},\boldsymbol{q}_{t}^{\mathsf{PV}})$

• Dynamic equations:

$$\mathsf{D}_{t}^{\theta}(\mathbf{x}_{t-1}, \mathbf{u}_{t}, \mathbf{q}_{t}^{\mathsf{PV}}) = \begin{cases} s_{t}^{j} = s_{t-1}^{j} - d_{t}^{j} + \sum_{i} u_{t}^{ij} & \forall j \\ SOC_{t} = SOC_{t-1} - \rho\phi_{t}^{-} + \rho\phi_{t}^{+} \end{cases}$$

• Initial conditions : $s_0 = 0 SOC_0 = SOC_{min}$

Stochastic parametrized operational problem

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s.c. $x_{t} = \mathsf{D}_{t}^{\theta}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t}, \boldsymbol{q}_{t}^{\mathsf{PV}}) \qquad x_{t} \in X_{t}^{\theta} \qquad \forall t \in [T]$

• State variables' feasible domain:

$$X_{t}^{\theta} = \begin{cases} 0 \leq s_{t}^{j} \leq s_{max}^{j} & \forall j \in J \\ SOC_{min} \leq SOC_{t} \leq SOC_{max} \end{cases}$$

Stochastic parametrized operational problem

$$(\mathsf{P}_{\theta}) \min_{(\boldsymbol{u}_{t}, \boldsymbol{x}_{t})_{t \in [T]}} \mathbb{E} \left[\sum_{t=1}^{T} \mathcal{L}_{t}^{\theta}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t}, \boldsymbol{q}_{t}^{\mathsf{PV}}) \right]$$

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$$u_{t} \in \mathcal{U}_{t}^{\theta}(\boldsymbol{x}_{t-1}, \boldsymbol{q}_{t}^{\mathsf{PV}}) \subset \mathcal{U}_{t}^{\theta} \qquad \forall t \in [T]$$

• Feasible domain of controls:

$$\mathcal{U}_{t}^{\theta}(\mathbf{x}_{t-1}, \mathbf{q}_{t}^{\mathsf{PV}}) = \begin{cases} \mathbf{b}_{t}^{ij} \in \{0, 1\} & \forall i \in I, j \in J \\ u_{\min}^{ij} \mathbf{b}_{t}^{ij} \leq u_{t}^{ij} \leq u_{\max}^{ij} \mathbf{b}_{t}^{ij} & \forall i \in I, j \in J \\ \mathbf{q}_{t}^{\mathsf{grid}}, \mathbf{v}_{t}^{\mathsf{ID}}, \phi_{t}^{+}, \phi_{t}^{-} \geq 0 \\ \phi_{t}^{+} \leq \phi_{\max}^{+}, \phi_{t}^{-} \leq \phi_{\max}^{-} \\ \dots \end{cases}$$

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• Controls constraints:

$$\mathcal{U}_{t}^{\theta}(\mathbf{x}_{t-1}, \mathbf{q}_{t}^{\mathsf{PV}}) = \begin{cases} \dots \\ \sum_{j} b_{t}^{ij} \leq 1 & 1 \text{ product per mill} \\ \max_{i} b_{t}^{ia} + \max_{i} b_{t}^{ic} \leq 1 & \text{Shared resources} \\ q_{t}^{\mathsf{load}} \leq q_{t}^{\mathsf{grid}} + q_{t}^{\mathsf{PV}} + \phi_{t}^{-} - \phi_{t}^{+} & \text{Load balance} \\ q_{t}^{\mathsf{grid}} = v_{t}^{\mathsf{DA}} + v_{t}^{\mathsf{ID}} & \text{Energy purchases} \end{cases}$$

Stochastic parametrized operational problem

$$(\mathsf{P}_{\theta}) \min_{(\boldsymbol{u}_{t}, \boldsymbol{x}_{t})_{t} \in [\mathcal{T}]} \mathbb{E} \left[\sum_{t=1}^{T} \mathcal{L}_{t}^{\theta}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t}, \boldsymbol{q}_{t}^{\mathsf{PV}}) \right]$$
s.c $\boldsymbol{x}_{t} = \mathsf{D}_{t}^{\theta}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{t}, \boldsymbol{q}_{t}^{\mathsf{PV}}) \quad \boldsymbol{x}_{t} \in X_{t}^{\theta} \quad \forall t \in [\mathcal{T}]$
 $\boldsymbol{u}_{t} \in \mathcal{U}_{t}^{\theta}(\boldsymbol{x}_{t-1}, \boldsymbol{q}_{t}^{\mathsf{PV}}) \subset \mathcal{U}_{t}^{\theta} \quad \forall t \in [\mathcal{T}]$
 $\sigma(\boldsymbol{u}_{t}) \subset \sigma(\boldsymbol{q}_{1}^{\mathsf{PV}}, ..., \boldsymbol{q}_{t}^{\mathsf{PV}}) \quad \forall t \in [\mathcal{T}]$

- Objective: we minimize the expected cost over $[1, \ldots, T]$;
- Instantaneous cost: $L_t^{\theta}(\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{q}_t^{\mathsf{PV}}) := p_t^{\mathsf{ID}} v_t^{\mathsf{ID}};$
- Non-anticipativity constraints: we don't know what happens in the future (after *t*).

2 Problem description

Operational problem solution methods The (corrected) Expected Value Strategy Model Predictive Control Dynamic programming The Look-Ahead Strategy

4 Design problem solution methods

5 Numerical Results

6 Conclusion

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Figure: Deterministic procedure to adapt variables to uncertainties.

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$$x_{t'} = D_{t}(x_{t'-1}, u_{t'}) \in \mathbb{X}_{t'}$$
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2 Problem description

3 Operational problem solution methods

The (corrected) Expected Value Strategy Model Predictive Control

Dynamic programming

The Look-Ahead Strategy

4 Design problem solution methods

5 Numerical Results

6 Conclusion

Operational problem (P_{θ})

Large multistage stochastic mixedinteger problem

DYNAMIC PROGRAMMING




 $V_t(x)$: optimal expected cost on $\llbracket t, T \rrbracket$ from state x















Introduction

Problem description

Operational problem solution methods

The (corrected) Expected Value Strategy Model Predictive Control Dynamic programming The Look-Ahead Strategy

4 Design problem solution methods

5 Numerical Results

6 Conclusion









1 Introduction

2 Problem description

Operational problem solution methods

4 Design problem solution methods The approach The 2-stage strategy

6 Numerical Results

6 Conclusion

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$$\bigcup_{\substack{ \mathsf{Design problem} \\ \mathsf{Find the optimal } \theta}}$$





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- 1. Expected Value strategy: solves a deterministic version of the whole problem to determine θ ;
- 2. 2-stage strategy: takes decision θ minimizing the expected cost over S_{MC} scenarios;
- 3. Stochastic Dual Dynamic Programming: solves the continuous relaxation of the problem.

1 Introduction

Problem description

Operational problem solution methods

Design problem solution methods The approach

The 2-stage strategy

6 Numerical Results

6 Conclusion

First Stage

Determining θ































ξ1 ξ2 ξ3

1-stage problem.













- 1 Introduction
- 2 Problem description
- Operational problem solution methods
- ④ Design problem solution methods

5 Numerical Results Operational Problem results Design Problem results


- Prices given by the Korea Electricity Power Corporation (KEPCO) website;
- Data collection for solar generation on NEDO website;
- Various renewable size, a factor $F \in \{0.5, 1, 2, 3\}$;
- Various battery sizing: *SOC_{max}* represents 0.5, 3 or 6 hours of maximum renewable production;
- Final demand $d_T^j > 0$.

Anticipative Regret (AR)

For a strategy ψ , and a scenario $\xi_{[T]}$,

$$AR^{\psi}(\xi_{[T]}) = \frac{\hat{V}^{\psi}(x_{0},\xi_{[T]};\theta) - \hat{V}^{\psi_{ant}}(x_{0},\xi_{[T]};\theta)}{|\hat{V}^{\psi_{ant}}(x_{0},\xi_{[T]};\theta)|}$$

SOC _{max}	0.5h			3h			6h		
Solar factor	L-A	MPC	EV	L-A	MPC	EV	L-A	MPC	EV
0.5	4.9	0.5	1.0	6.1	0.5	2.4	5.4	0.5	3.2
1.0	6.1	1.3	4.6	3.9	0.9	6.3	2.4	0.6	6.4
2.0	8.7	3.9	14	4.5	1.5	15	4.0	1.4	15
3.0	11	5.6	27	9.1	3.6	28	8.2	3.5	28

Table: Anticipative Regret (AR) in % for different methods (EV strategy, MPC, Look-ahead) for the operational problem: MPC yields the most satisfactory results.

- 1 Introduction
- 2 Problem description
- Operational problem solution methods
- ④ Design problem solution methods
- 5 Numerical Results Operational Problem results Design Problem results



		OPT			AR (in %)	
Solar Factor	MPC	2stage	SDDP	MPC	2stage	SDDP
0.5	6067	6023	6038	1.6	0.9	1.1
1.0	5471	5483	5451	2.1	2.3	1.7
2.0	4552	4553	4481	4.2	4.2	2.5
3.0	3714	3691	3641	8.7	7.9	6.7

Table: Expected Cost (Opt) and Anticipative Regret (AR) for different methods (EV, 2–stage, SDDP) determining θ and then MPC.

IN A NUTSHELL

- We decompose an industrial energy-aware problem into an operational problem embedded in a design problem.
- We confront methods relaxing either integrity or information constraints.
- For these kinds of problems:
 - Considering uncertainties is relevant;
 - If uncertainties impact future costs, a stochastic method yields better results.

Future works

- Find a satisfactory stochastic heuristic dealing with binary variables;
- Incorporate energy market vision.

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