Factory Production and Energy Supply Planning with Multistage Stochastic Optimization

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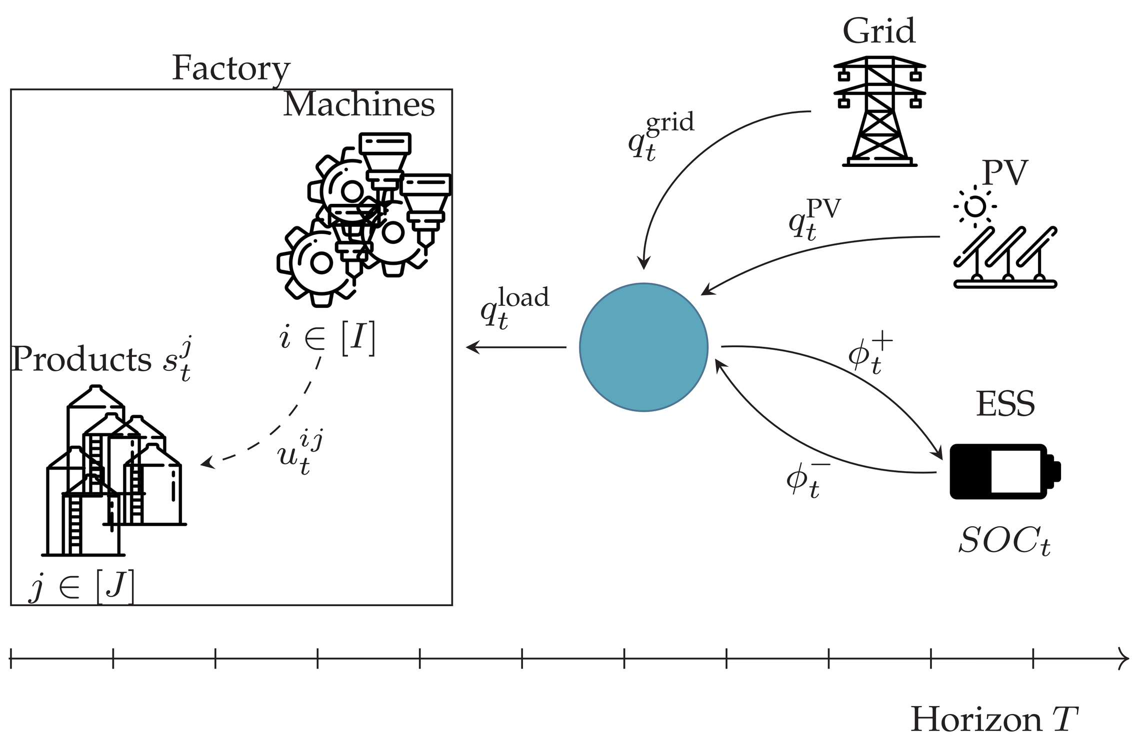
Motivations: Study Case



- **Problem** We must provide the factory with a **production and energy supply plan** minimizing energy costs over a time horizon ($T = 24$). Energy is bought in real time or in advance.
- **Variables**
  Continuous: production, stocks, energy bought, ESS charge/discharge and stocks;
  Binary: $b_t^{ij} = \mathbb{1}_{\{j \text{ produced by } i \text{ at } t\}}$
- **Constraints**
  Dynamic Equations for stock variables;
  Shared resources: hard constraints (binary).
  Demand: for each product.

Mathematical Model

We consider a multistage stochastic problem:

$$
\text{(P)} \quad \min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{b}} \ \mathbb{E}\left[\sum_{t=1}^{T} L_t(\boldsymbol{x_{t-1}}, \boldsymbol{u_t}, \boldsymbol{b_t}, \boldsymbol{\xi_t})\right]
$$

$$
\begin{aligned}
\boldsymbol{x_{t+1}} &= F_{t+1}(\boldsymbol{x_t}, \boldsymbol{u_t}, \boldsymbol{b_t}, \boldsymbol{\xi_t}) && \forall t\\
\boldsymbol{u_t} &\in \mathcal{U}(\boldsymbol{x_t}, \boldsymbol{\xi_t}) \subset \mathbb{R}^{n_u} && \forall t\\
\boldsymbol{b_t} &\in \mathcal{B}(\boldsymbol{x_t}, \boldsymbol{\xi_t}) \subset \{0,1\}^{n_b} && \forall t\\
\sigma(\boldsymbol{u_t}, \boldsymbol{b_t}) &\subset \sigma(\boldsymbol{\xi_1}, \dots, \boldsymbol{\xi_t}) && \forall t
\end{aligned}
$$

Let $(\boldsymbol{\xi}_t)_{t\in[T]}$ be a sequence of finitely supported **random variables**. We define a **scenario** $(\xi_t^{j t})_{t\in[T]}$ as a realization of $(\boldsymbol{\xi}_t)_{t\in[T]}$, and the **scenario tree** $\mathcal{T}$ as the collection of all scenarios.

We denote $\mathcal{N}_t$ the set of nodes in $\mathcal{T}$ of depth t. Then a **node** $\nu \in \mathcal{N}_t$ reads its ancestor information from the root:

$$
\nu = (a(\nu), \xi_t^{k_t}) = ((\dots(\underbrace{(\emptyset, \xi_1^{k_1})}_{\in \mathcal{N}_1}, \xi_2^{k_2}), \dots, \xi_{t-1}^{k_{t-1}}), \xi_t^{k_t})
$$

We can reformulate the problem in its extensive form:

$$
(\text{P}_{\text{ext}}) \quad \min_{x_\nu, u_\nu, b_\nu} \sum_{t=1}^{T} \sum_{\nu \in \mathcal{N}_t} \pi_\nu L_t(x_\nu, u_\nu, b_\nu, \xi_\nu)
$$

$$
\begin{aligned}
x_\nu &= F_\nu(x_{a(\nu)}, u_{a(\nu)}, b_{a(\nu)}, \xi_{a(\nu)}) && \forall \nu\\
u_\nu &\in \mathcal{U}(x_\nu, \xi_\nu) && \forall \nu\\
b_\nu &\in \mathcal{B}(x_\nu, \xi_\nu) \subset \{0,1\}^{n_b} && \forall \nu
\end{aligned}
$$

Dynamic Programming

- Under **stage-wise independence** of the noises, we reformulate the problem with **dynamic equations**.
- Because the state variables are continuous, classic dynamic programming is limited by the **curse of dimensionality** and is not a reasonable option here.
- If the problem was continuous, SDDP would solve it easily. Unfortunately we have **binary variables**, modeling hard constraints.

Branch-and-Bound

- The extensive MILP (P_{ext}) is **intractable** and cannot be solved with classical solvers.
- We would like to reduce the problem to multiple continuous sub-problem by resorting to **branch-and-bound methods**.
- However, even the relaxed problem are too big to be solved by LP solver ($O(10^{24})$ variables).
  - ➥ Can we use SDDP to solve them?
  - ➥ We introduce **assignation function**:

$$
\mathfrak{b} : \mathcal{T} \to \{\{0\}, \{1\}, \{0,1\}, [0,1]\}^{n_b}.
$$

State of the Art

- **Model Predictive Control**

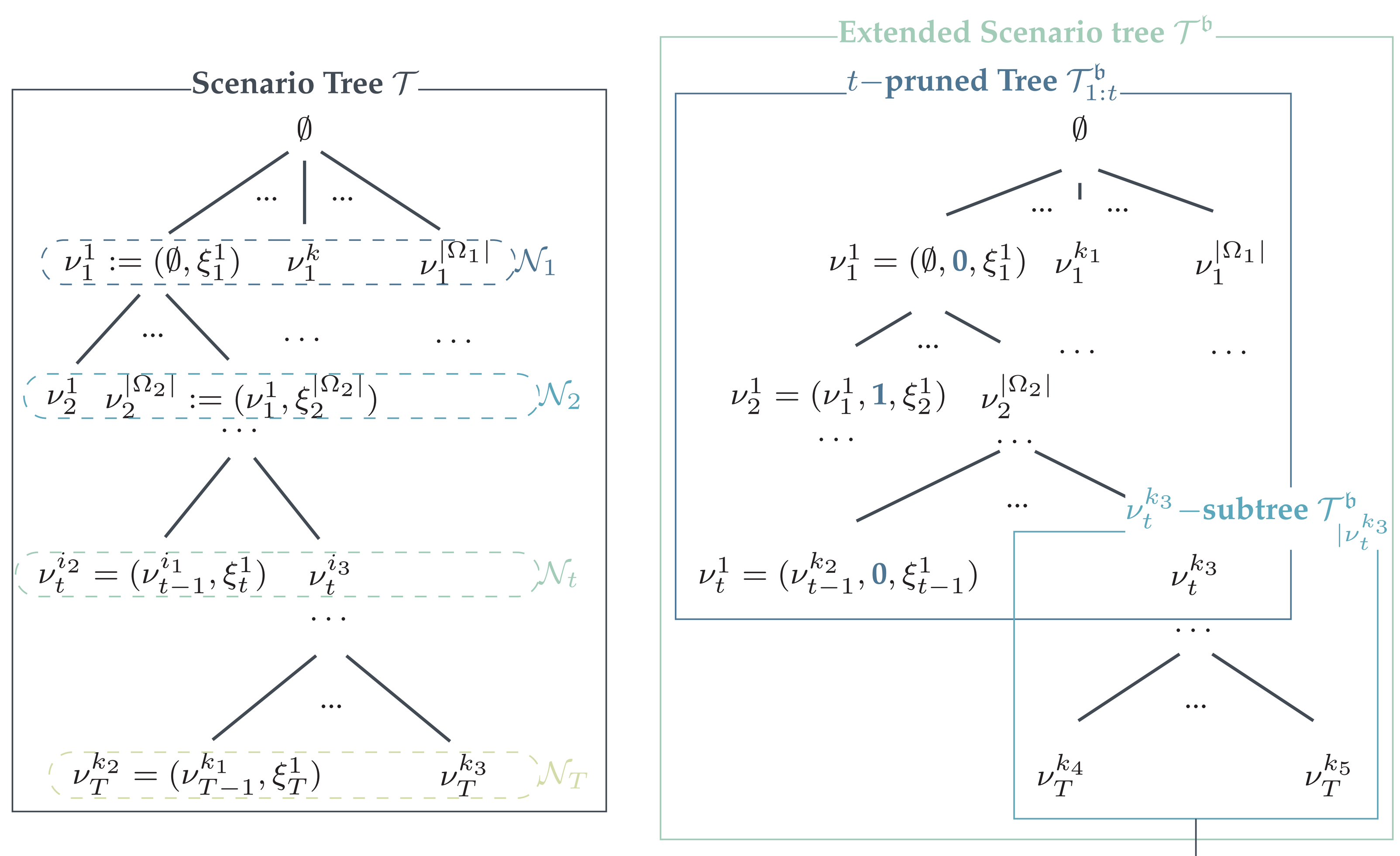
  Principle: solve deterministic sub-problems, adjusting trajectory as random realizations are revealed.
  Pros:use of deterministic solvers, no stagewise independence.
  Cons: no solution quality guarantee, slow online running time.
- **Stochastic Dynamic Programming**

  Principle: with stagewise independence, we solve the problem with dynamic equations.
  Pros: few assumptions, easily implemented
  Cons: curse of dimensionality.
- **Stochastic Dual Dynamic Programming**

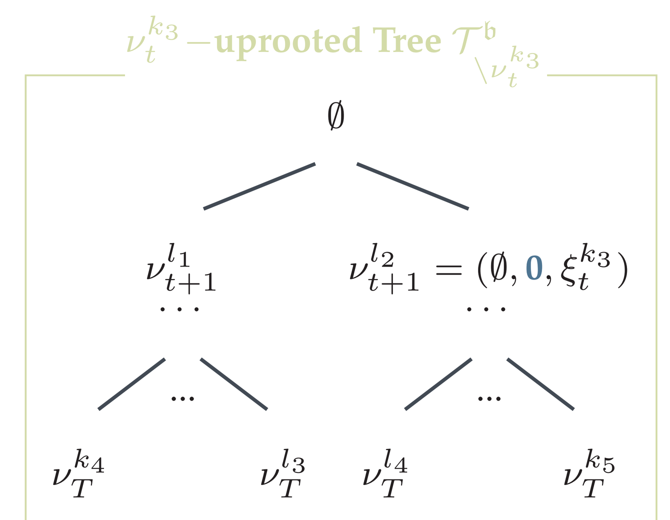
  Principle: solves continuous multistage linear stochastic problems by constructing Benders-like cuts.
  Pros: fast in practice, and theoretical guarantee.
  Cons: cannot handle integer variables.
- **Stochastic Dual Dynamic integer Programming**

  Principle: algorithm built on SDDP to solve multistage linear stochastic problems with only binary state variables.
  Pros: Theoretical guarantees.
  Cons: Slow iterations and convergence.

Extended Scenario Tree



- We consider an assignation function $\mathfrak{b}$ as part of the noise and integrate it in the **extended scenario tree** $\mathcal{T}^{\mathfrak{b}}$.
- $\mathcal{T}^{\mathfrak{b}}$ is $t-$**independent** if $\mathcal{T}^{\mathfrak{b}}_{\setminus \nu_1} = \mathcal{T}^{\mathfrak{b}}_{\setminus \nu_2}$ $\forall \nu_1, \nu_2 \in \mathcal{N}_t$
- We define $(\text{P}^{\mathfrak{b}}_{\tau:T})$ as the sub-problem on horizon $[\![\tau, \dots, T]\!]$ with assignation function $\mathfrak{b}$.



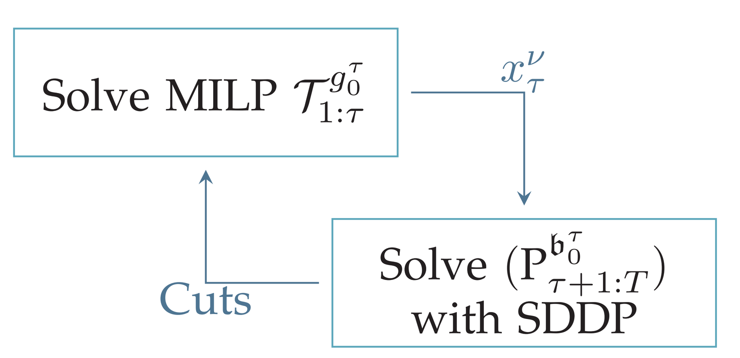
Proposition: Assume $\mathfrak{b}$ is such that $\mathfrak{b}(\nu) \neq \{0,1\}$ $\forall \nu$.
Then $\mathcal{T}^{\mathfrak{b}}$ is $\tau-$independent, and $(\text{P}^{\mathfrak{b}}_{\tau})$ can be solved with SDDP.

Association Function Example

We consider the assignation function $\mathfrak{b}_0^\tau$ that keeps binary constraints on sub-horizon $[\![1, \tau]\!]$ and relaxes those constraints on the rest of the horizon.

- ➥ scenario tree $\mathcal{T}^{\mathfrak{b}_0^\tau}$ is $\tau-$independent, and we can run SDDP on $(\text{P}^{\mathfrak{b}_0^\tau}_{\tau+1:T})$
- ➥ We can use SDDP cuts to represent the extensive sub-problem final cost function on horizon $[\![1, \tau]\!]$

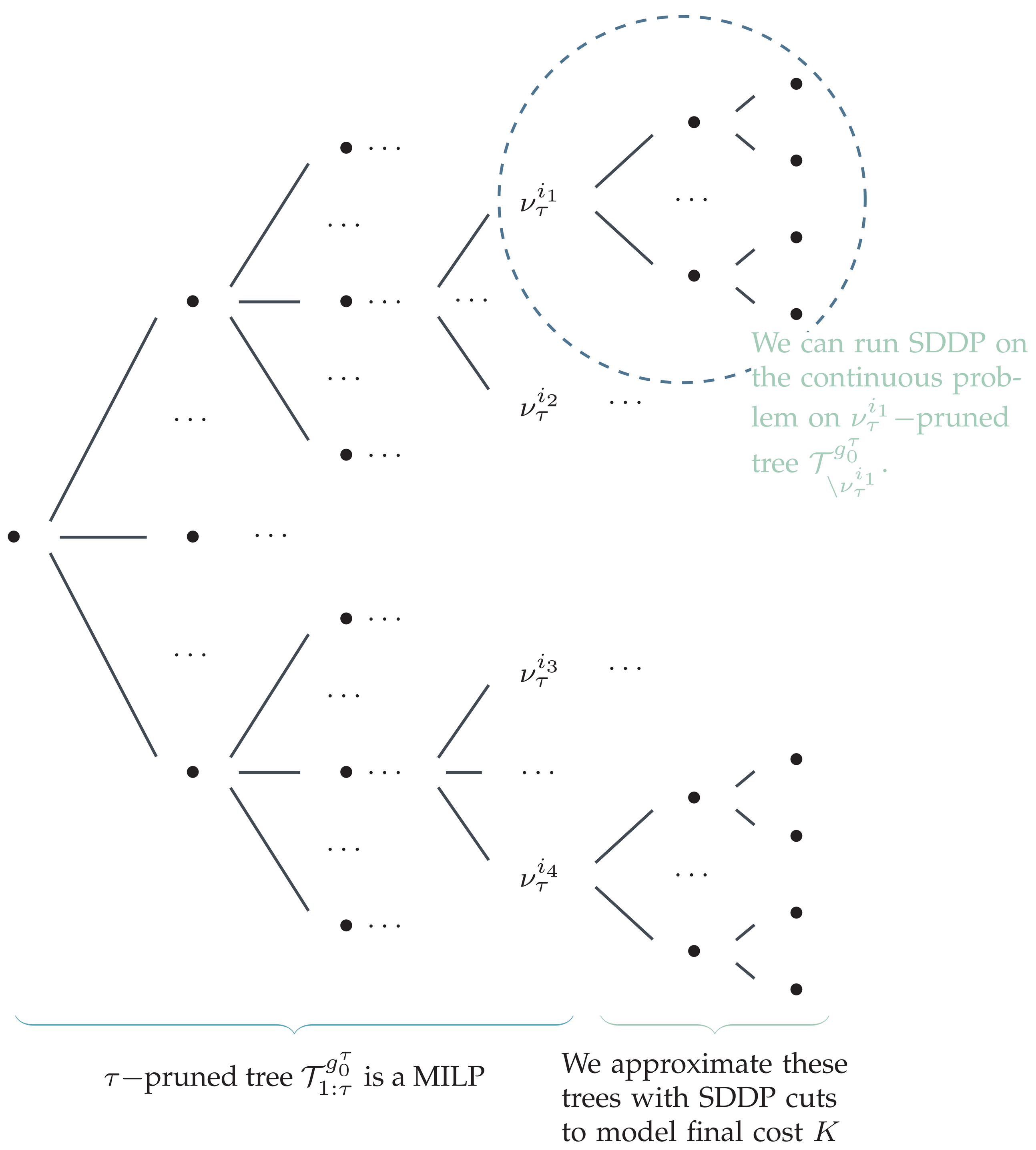
Algorithmic scheme:



➥ **Relaxed Horizon Heuristic**, we can compare its results with MPC.

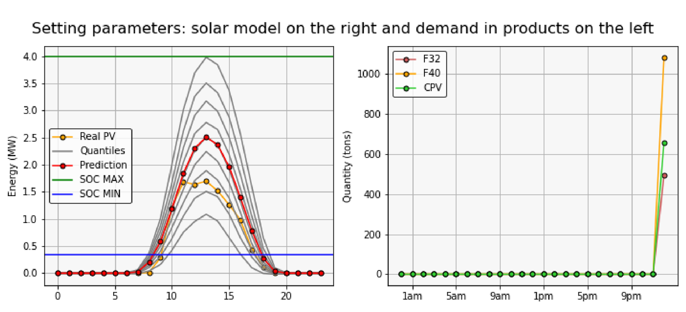
Future works:

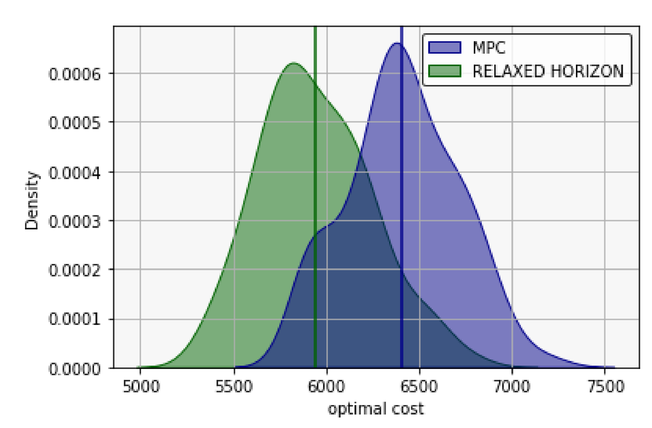
- Study convergence;
- Exploit specific problem structure (*e.g.* minimal up/down time) to find other compatible assignation functions.



Numerical Results

We solve the problem with both intraday and day-ahead markets, and simulate in a rolling horizon. At each step, we compute decisions by solving $(\text{P}_{t:T})$ either with MPC, or with the relaxed horizon heuristic ($\tau = 2$).





References

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