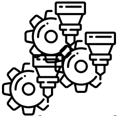
Coupled industrial production and energy supply planning
ROADEF

Zoé Fornier
November 2021

Contents

Mills



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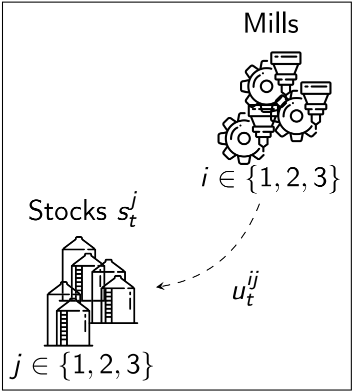
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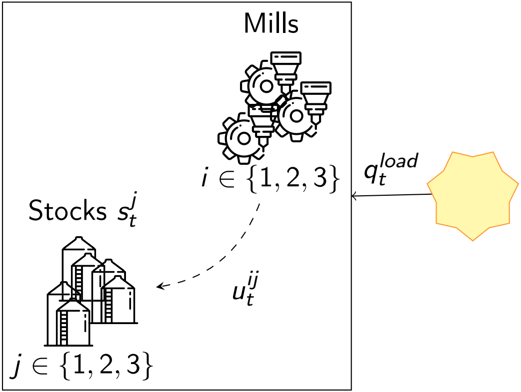
u_t^{ij}
$j \in \{1, 2, 3\}$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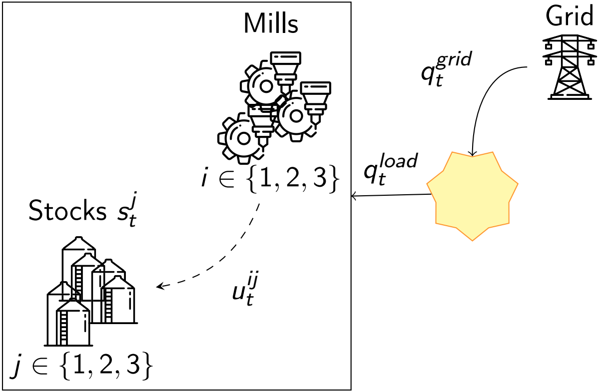
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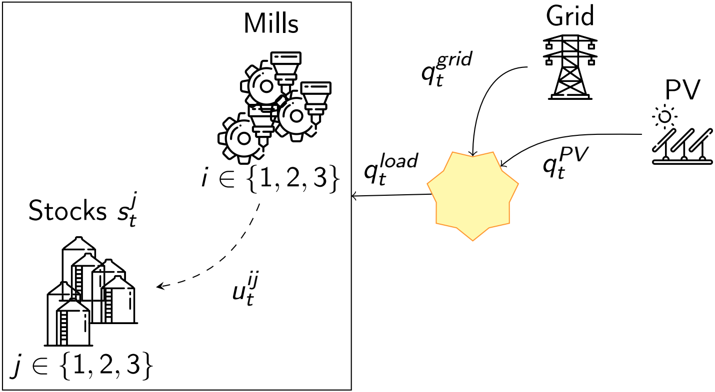
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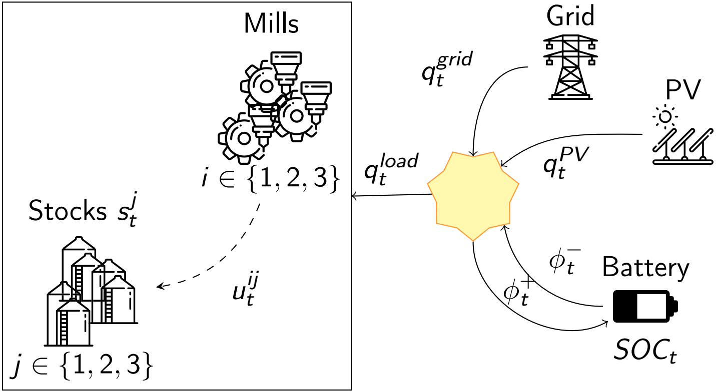
PV
q_t^{PV}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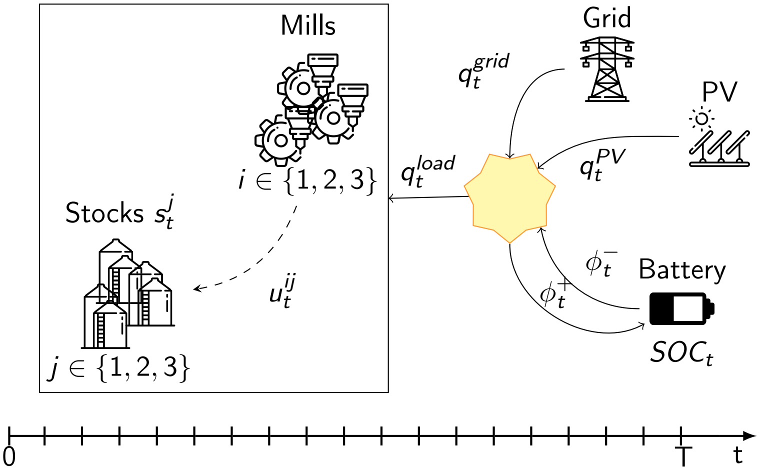
q_t^{PV}
ϕ_t^-
ϕ_t^+
Battery
SOC_t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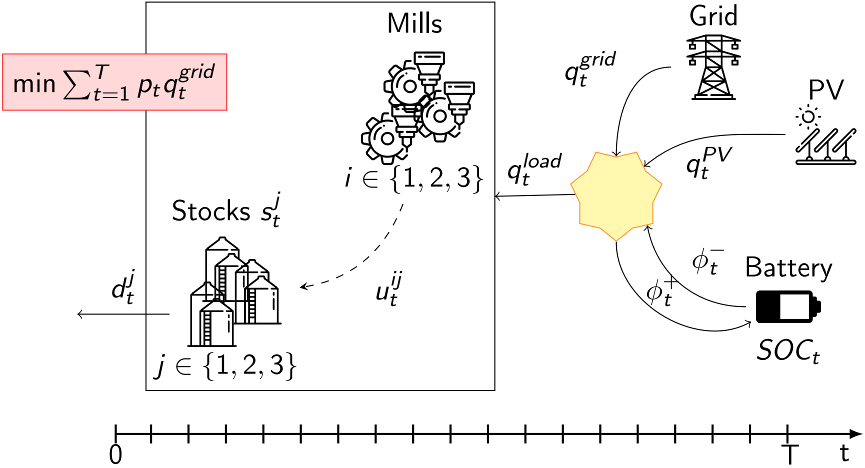
$\min \sum_{t=1}^{T} p_t q_t^{grid}$
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$i \in \{1, 2, 3\}$
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$j \in \{1, 2, 3\}$
Grid
q_t^{grid}
PV
q_t^{PV}
q_t^{load}
ϕ_t^-
ϕ_t^+
Battery
SOC_t
0
T
t

Contents

Deterministic formulation

$$\min \quad \sum_{t=1}^{T} L_t(x_{t-1}, u_t, q_t^{PV})$$

- State variables: $x_t := (SOC_t, s_t^1, s_t^2, s_t^3)$
- Controls: $u_t := (\underbrace{q_t^{grid}, \phi_t^+, \phi_t^-, (u_t^{ij})_{i,j\in[3]}}_{\in\mathbb{R}^+}, \underbrace{(b_t^{ij})_{i,j\in[3]}}_{\in\{0,1\}})$
- Instantaneous cost : $L_t(x_{t-1}, u_t, q_t^{PV}) := p_t q_t^{grid}$

Deterministic formulation

$$\min \quad \sum_{t=1}^{T} L_t(x_{t-1}, u_t, q_t^{PV})$$

$$\text{s.c} \quad x_t = D_t(x_{t-1}, u_t), \qquad x_0 \text{ fixed} \qquad \forall t \geq 1$$

- Dynamic equations:

$$D_t(x_{t-1}, u_t) = \begin{cases} s_t^j = s_{t-1}^j - d_t^j + \sum_i u_t^{ij} & \forall j \\ SOC_t = SOC_{t-1} - \frac{1}{\rho^-}\phi_t^- + \rho^+\phi_t^+ \end{cases}$$

- Initial conditions : $s_0^j = 0 \ \forall j, \ SOC_0 = SOC_{min}$

Deterministic formulation

$$\begin{aligned} \min \quad & \sum_{t=1}^{T} L_t(x_{t-1}, u_t, q_t^{PV}) \\ \text{s.c} \quad & x_t = D_t(x_{t-1}, u_t), \qquad x_0 \text{ fixed} \qquad \forall t \geq 1 \\ & u_t \in \mathbb{U}_t(x_t, q_t^{PV}) \end{aligned}$$

- Feasible domain of controls:

$$\mathbb{U}_t(x_t, q_t^{PV}) = \begin{cases} b_t^{ij} \in \{0, 1\} & \forall i, j \\ u_{min}^{ij} b_t^{ij} \leq u_t^{ij} \leq u_{max}^{ij} b_t^{ij} & \forall i, j \\ q_t^{grid}, \phi_t^+, \phi_t^- \geq 0 & \\ \phi_t^+ \leq \phi_{max}^+ \quad \phi_t^- \leq \phi_{max}^- & \\ \dots & \end{cases}$$

Deterministic formulation

$$\min \quad \sum_{t=1}^{T} L_t(x_{t-1}, u_t, q_t^{PV})$$

$$\text{s.c} \quad x_t = D_t(x_{t-1}, u_t), \qquad x_0 \text{ fixed} \qquad \forall t \geq 1$$

$$u_t \in \mathbb{U}_t(x_t, q_t^{PV})$$

- Controls constraints:

$$\mathbb{U}_t(x_t, q_t^{PV}) = \begin{cases} \dots \\ \sum_j b_t^{ij} \leq 1 & \text{1 product per mill} \\ \max_i b_t^{i1} + \max_i b_t^{i3} \leq 1 & \text{Incompatibility} \\ q_t^{load} \leq q_t^{grid} + q_t^{PV} + \phi_t^- - \phi_t^+ & \text{Load balance} \end{cases}$$

Deterministic formulation

$$\begin{aligned} \min \quad & \sum_{t=1}^{T} L_t(x_{t-1}, u_t, q_t^{PV}) \\ \text{s.c} \quad & x_t = D_t(x_{t-1}, u_t), \qquad x_0 \text{ fixed} \qquad \forall t \geq 1 \\ & u_t \in \mathbb{U}_t(x_t, q_t^{PV}), \qquad x_t \in \mathbb{X}_t \qquad \forall t \geq 1 \end{aligned}$$

- State variables' feasible domain:

$$\mathbb{X}_t = \begin{cases} 0 \leq s_t^j \leq s_{max}^j & \forall j \\ SOC_{min} \leq SOC_t \leq SOC_{max} \end{cases}$$

Contents

NP-completeness

- L production lines and C cements; no incompatibility

Proposition 1

Given a set of parameters, and K $\in \mathbb{R}$: is there a solution whose cost is inferior to K ?
The corresponding decision problem is NP-complete.

A polynomial case

- L production lines and C cements

Proposition 2

Given a set of parameters: is there a feasible solution ?
If C is fixed, and all u^{ij}_{max} are independent from i, the corresponding decision problem is Polynomial.

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Contents

Stochastic formulation : Hazard Decision

$$\min \quad \mathbb{E}\left[\sum_{t=1}^{T} L_t(\boldsymbol{x_{t-1}}, \boldsymbol{u_t}, \boldsymbol{q_t^{PV}})\right]$$

$$\text{s.c} \quad \boldsymbol{x_t} = D_t(\boldsymbol{x_{t-1}}, \boldsymbol{u_t}), \qquad x_0 \text{ fixed} \qquad \forall t \geq 1$$

$$\boldsymbol{u_t} \in \mathbb{U}_t(\boldsymbol{x_t}, \boldsymbol{q_t^{PV}}), \qquad \boldsymbol{x_t} \in \mathbb{X}_t \qquad \forall t \geq 1$$

$$\underbrace{\sigma(u_t) \subset \sigma(\boldsymbol{q_1^{PV}}, ..., \boldsymbol{q_t^{PV}})}_{\text{non-anticipativity constraints}}$$

- $(\boldsymbol{q_t^{PV}})_{t\in[T]}$ are random variables: assumed **independent**
- We minimize the expected cost
- We don't know what happens in the future (after t)

Contents

Algorithm 1: Model predictive control

Input: x_0, $\hat{q}^{PV}$ solar prediction for the whole horizon

for t : 1,...,T **do**

Observe q_t^{PV} realization of solar energy at the current time, and using the prediction for the future, solve the following **deterministic** subproblem:

$$(u_{t'}^{\sharp})_{t' \geq t} = \underset{u_t, (u_{t'})_{t'>t}}{\arg\min} \; L_t(x_{t-1}, u_t, q_t^{PV}) + \sum_{t'=t+1}^{T} L_t(x_{t'-1}, u_{t'}, \hat{q}_{t'}^{PV})$$

$$x_{t'} = D_t(x_{t'-1}, u_{t'}) \in \mathbb{X}_{t'}$$

$$u_{t'} \in \mathbb{U}_{t'}(x_{t'}, \hat{q}_{t'}^{PV})$$

$$x_t = \quad D_t(x_{t-1}, u_t^{\sharp})$$

This method will be our reference model.

Contents

- Define the Bellman's functions $V_t(x)$, a.k.a *cost-to-go* function, as the expected optimal cost on $[t, \dots, T]$ starting from x.
- $V_t(x)_{t \in [T]}$ can be computed recursively backward.
- $V_t(x)_{t \in [T]}$ cannot usually be exactly computed (e.g. when there is a continuous number of possible state x), therefore we must consider interpolations and approximations methods:
  - **discretization** of the space set $\boldsymbol{x_t^D} \in \mathbb{X}_{\boldsymbol{N}}$
  - **convex interpolation** of $(x_{t-1}^D, v_t^{x_{t-1}^D})$ to approximate V_t

Algorithm 2: Stochastic dynamic programming

```
// Initialization
```
$V_{T+1} := 0$

```
// Backward phase
```
for *t : T, T-1,...,1* **do**

  for $x^D_{t-1} \in \mathbb{X}_N$ **do**

    for ω *realization of* $\boldsymbol{q_t^{PV}}$ **do**

$$v_t^{x^D_{\boldsymbol{t-1}}} \mathrel{+}= p_\omega \min_{\boldsymbol{u_t} \in \mathbb{U}_t(\boldsymbol{y}, \boldsymbol{q_t^{PV}})} \left\{ L_t(\boldsymbol{x^D_{t-1}}, \boldsymbol{u_t}, \omega) + \tilde{V}_{t+1}(\boldsymbol{y}) \right\}$$

$$\boldsymbol{y} = F_t(\boldsymbol{x^D_{t-1}}, \boldsymbol{u_t})$$

  Define $\tilde{V}_{t+1}$ convex interpolation of $(x^D_t, v^{x^D_t}_{t+1})_{x^D_t \in \mathbb{X}_N}$

Curse of dimensionality: $\mathcal{O}(T.N^4.|\Omega|_t)$ MILP to solve.

- **Stochastic Dual Dynamic Programming** (SDDP) is a classic algorithm which solves continuous multistage linear stochastic problems.

- The algorithm consists in a fixed number of successive steps involving:
  - a forward phase: we **randomly draw** $(\xi_t^k)_t$, a scenario, on which we compute the optimal **trajectory** $(x_t^k)_{t\geq 0}$ for the strategy given by the current cost-to-go approximation.
  - a backward phase: we **refine our** V_t^{k-1} **approximation** by constructing **new cuts** (similar to Benders) from x_{t-1}^k.

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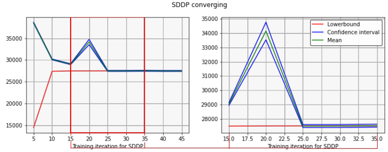

Heuristic based on SDDP

- We consider offline time (construction of a decision strategy) and online time (simulation).
- Compared to DP and MPC, SDDP is very fast.
- However, SDDP solves a convex continuous relaxation of the problem.
- We aim at finding a hybrid method which takes advantage of SDDP's fast computational time, and returns a feasible solution (with binary constraint).

Contents

Cuts Heuristic

Main Idea : estimate V_t using SDDP cuts.

- SDDP **estimates V_t^r, the continuous relaxation of V_t**
- Given a solar energy scenario $(\omega_1, \dots, \omega_T)$, we compute :

$$u_t^\sharp := \underset{u_t \in \mathbb{U}_t(y,\omega_t)}{\arg\min} \; L_t(x_{t-1}^\sharp, u_t, \omega_t) + \boldsymbol{V_{t+1}^r(y)}$$

$$y = F_t(x_{t-1}^\sharp, u_t)$$

$$x_t^\sharp := F_t(x_{t-1}^\sharp, u_t^\sharp)$$

Pros: the offline time amounts to SDDP computation time, the online time is similar to dynamic programming online time.

Cons: the coarse approximation used may be inadequate to get feasible solutions.

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Main Idea: improving DP using 2-stage problems.

- DP reduces a T-stage problem to T consecutive 1-stage problems.
- SDDP solves a convex continuous relaxation.
- Instead, let's consider $T-1$ consecutive **2-stage problems**:
  - **non-anticipativity** constraint, we need to consider all realizations at $t+1$ and the corresponding decision variables ($\times|\Omega|_t$)
  - we minimize the sum of
    - instantaneous cost at t,
    - expected instantaneous cost at $t+1$,
    - and cost-to-go from $t+2$.

Pros : better handling of the binary variables.

Cons : the problem size grows significantly.

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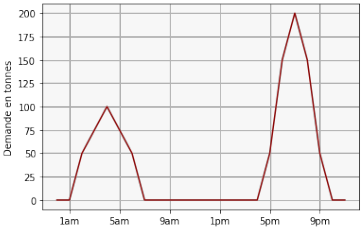
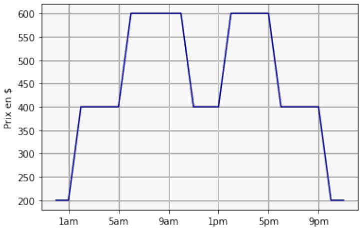
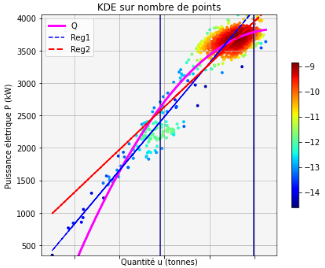
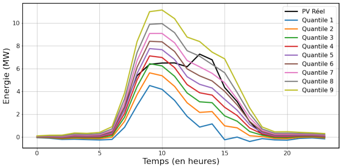
Fitting parameters to data









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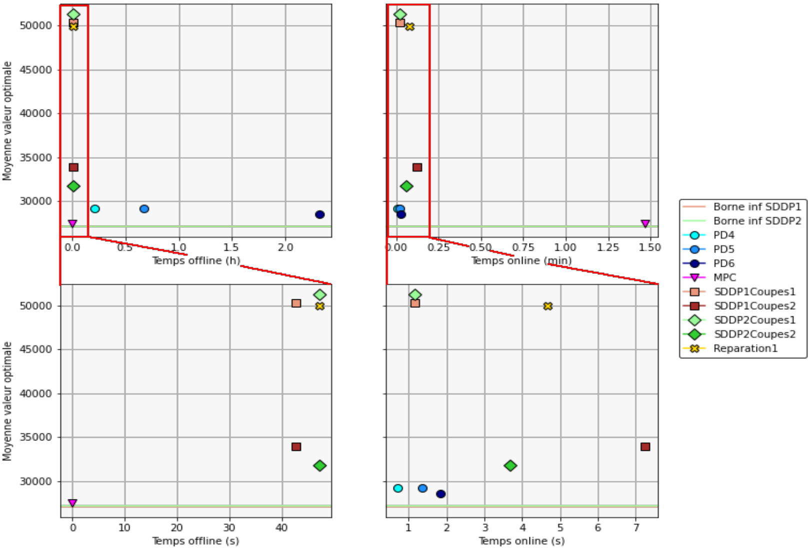
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2.0
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0.50
0.75
1.00
1.25
1.50
Temps online (min)
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10
20
30
40
Temps offline (s)
1
2
3
4
5
6
7
Temps online (s)
Borne inf SDDP1
Borne inf SDDP2
PD4
PD5
PD6
MPC
SDDP1Coupes1
SDDP1Coupes2
SDDP2Coupes1
SDDP2Coupes2
Reparation1

In a nutshell:

- we studied the complexity of the **deterministic** problem;
- we test two classical methods on the stochastic problem: DP (with convex interpolation) and MPC;
- finally, we developed various **heuristics** taking advantage of SDDP for speed and multi-step Bellman equation for better handling of binary variables.

Futur works:

- incorporate day-ahead problem;
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